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publication

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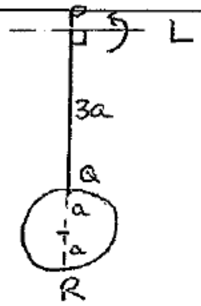
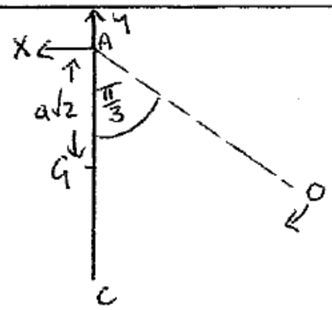
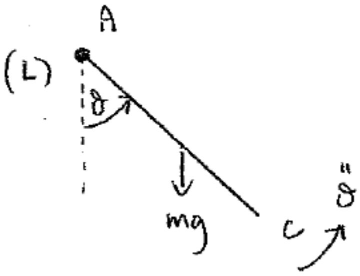
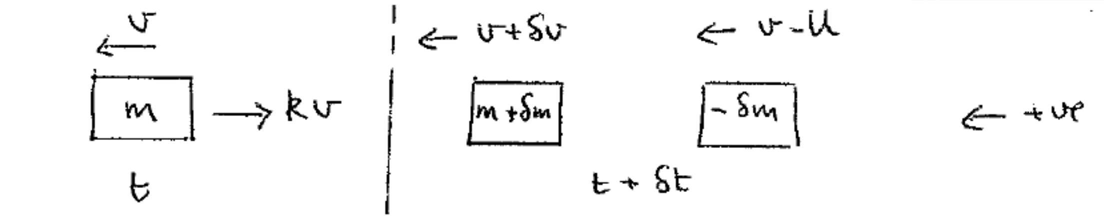
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	$\lvert 2\mathbf{i}+\mathbf{j}+2\mathbf{k}\rvert = 3 \Rightarrow \mathbf{F}_1 = 6\mathbf{i}+3\mathbf{j}+6\mathbf{k}$	B1
	$\lvert \mathbf{i}+8\mathbf{j}-4\mathbf{k}\rvert = 9 \Rightarrow \mathbf{F}_2 = 2\mathbf{i}+16\mathbf{j}-8\mathbf{k}$	B1
	$WD = (6\mathbf{i}+3\mathbf{j}+6\mathbf{k}+2\mathbf{i}+16\mathbf{j}-8\mathbf{k})\cdot(3\mathbf{i}+2\mathbf{j}-\mathbf{k}-\mathbf{i}-\mathbf{j}-\mathbf{k})$	M1 A1√ on F
	$= (8\mathbf{i}+19\mathbf{j}-2\mathbf{k})\cdot(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	M1
	$= 39\text{ J}$	A1 (6)
2.	$\text{I.F.} = e^{\int 2\,dt} = e^{2t}$	B1
	$\Rightarrow e^{2t}\dfrac{d\mathbf{r}}{dt} + 2e^{2t}\mathbf{r} = 3e^{t}\mathbf{j}$	M1
	$\Rightarrow \dfrac{d}{dt}(\mathbf{r}e^{2t}) = 3e^{t}\mathbf{j}$	
	$e^{2t}\mathbf{r} = 3e^{t}\mathbf{j}\ (+c)$	M1 A1
	$t=0,\ \mathbf{r} = 2\mathbf{i}-\mathbf{j} \Rightarrow 2\mathbf{i}-\mathbf{j} = 3\mathbf{j}+c$	M1
	$2\mathbf{i}-4\mathbf{j} = c$	A1
	$\Rightarrow \mathbf{r} = 3e^{-t}\mathbf{j} + (2\mathbf{i}-4\mathbf{j})e^{-2t}$	A1 (7)
3. a)	 $\mathbf{R} = \mathbf{F}_1+\mathbf{F}_2$	M1
	$\mathbf{R} = (-\mathbf{i}+3\mathbf{j}+2\mathbf{k})\text{N}$	A1 (2)
b)	$M(O):\ \begin{pmatrix}1\\1\\1\end{pmatrix}\times\begin{pmatrix}-1\\3\\2\end{pmatrix} + \begin{pmatrix}7\\-3\\8\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix}\times\begin{pmatrix}-1\\3\\2\end{pmatrix}$	M1 A2√ on R
	$\begin{pmatrix}-1\\-3\\4\end{pmatrix} + \begin{pmatrix}7\\-3\\8\end{pmatrix} = \begin{pmatrix}2y-3z\\-z-2x\\3x+y\end{pmatrix}$	A1 A1
	$\Rightarrow \begin{pmatrix}6\\-6\\12\end{pmatrix} = \begin{pmatrix}2y-3z\\-z-2x\\3x+y\end{pmatrix}$	
	Take $z=0$, one solution is $x=3, y=3, z=0$	M1
	$\therefore\ \mathbf{r} = \begin{pmatrix}3\\3\\0\end{pmatrix} + \lambda\begin{pmatrix}-1\\3\\2\end{pmatrix}$ is an equation of the line of action of $\mathbf{R}$	A1 (7)
		(9)

Question Number	Scheme	Marks
4 (a)	 DISC: $I_{diam} = \frac{1}{2}\left(\frac{1}{2}ma^2\right) = \frac{1}{4}ma^2$	M1 A1
	$I_L = \frac{1}{4}ma^2 + m(4a)^2$	M1 A1
	$= \frac{65}{4}ma^2$	B1
	ROD: $I_L = \frac{4}{3}\left(\frac{3a}{2}\right)^2 = 3ma^2$	
	$I_{TOTAL} = \frac{65}{4}ma^2 + 3ma^2 = \frac{77}{4}ma^2$ *	M1 A1 (7)
(b)	CAM: $\frac{77ma^2}{4}\omega = \left[\frac{77}{4}ma^2 + \frac{1}{2}m(4a)^2\right]\omega'$	M1 A1 A1
	$\Rightarrow \omega' = \frac{77}{109}\omega$	A1 (4)
		(11)
5 (a)	 Energy: $\frac{1}{2}\cdot\frac{8ma^2}{3}\dot{\theta}^2 = mga\sqrt{2}\left(1-\cos\frac{\pi}{3}\right)$	M1 A1
	$\Rightarrow \dot{\theta}^2 = \frac{3g\sqrt{2}}{8a}$	A1
	R↑: $Y - mg = ma\sqrt{2}\,\dot{\theta}^2$	M1 A1
	$= ma\sqrt{2}\cdot\frac{3g\sqrt{2}}{8a}$	M1
	$\Rightarrow Y = \frac{7mg}{4}$	A1 (7)
(b)	 M(L): $-mga\sqrt{2}\sin\theta = \frac{8ma^2}{3}\ddot{\theta}$	M1 A1 A1
	For small θ, $\sin\theta \simeq \theta$	
	$\Rightarrow -\frac{3g\sqrt{2}}{8a}\theta \simeq \ddot{\theta}$	M1
	Approx SHM	
	$\Rightarrow$ period: $2\pi\sqrt{\frac{8a}{3g\sqrt{2}}}$	A1 (5)
		(12)

Question Number	Scheme	
6(a)		
	Impulse - momentum:	M1 A3
	$-kv\,\delta t = (m+\delta m)(v+\delta v) + (-\delta m)(v-U) - mv$	
	$-kv\,\delta t = \cancel{mv} + m\delta v + \cancel{\delta m v} - \cancel{\delta m v} + \delta m U - \cancel{mv}$	
	$-kv = m\dfrac{\delta v}{\delta t} + U\dfrac{\delta m}{\delta t}$	
	limit as $\delta t \to 0$: $-kv = m\dfrac{dv}{dt} + U\dfrac{dm}{dt}$	
	$\dfrac{dm}{dt} = -\lambda \iff m = M - \lambda t$	B1
	$-kv = (M-\lambda t)\dfrac{dv}{dt} - \lambda U$	M1
	$\dfrac{dv}{dt} = \dfrac{\lambda U - kv}{M - \lambda t}$ $\quad *$	A1 (7)
(b)	$\displaystyle\int_0^t \frac{dt}{M-\lambda t} = \int_0^v \frac{dv}{\lambda U - kv}$	M1 A1
	$-\dfrac{1}{\lambda}\Big[\ln(M-\lambda t)\Big]_0^t = -\dfrac{1}{k}\Big[\ln(\lambda U - kv)\Big]_0^v$	A1
	$\dfrac{k}{\lambda}\big(\ln(M-\lambda t) - \ln M\big) = \ln(\lambda U - kv) - \ln \lambda U$	M1
	$\left(\dfrac{M-\lambda t}{M}\right)^{k/\lambda} = \dfrac{\lambda U - kv}{\lambda U}$	M1
	$v = \dfrac{\lambda U}{k}\left[1 - \left(1 - \dfrac{\lambda t}{M}\right)^{k/\lambda}\right]$ $\quad *$	A1 (6)
		(13)

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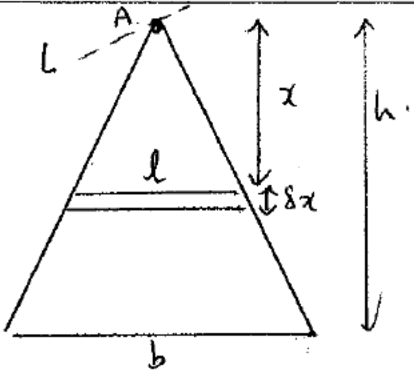
7. (a)



$\dfrac{l}{x} = \dfrac{b}{h} \quad \left(= \dfrac{2}{\sqrt{3}}\right)$ — M1

$\Rightarrow l = \dfrac{bx}{h} = \dfrac{2x}{\sqrt{3}}$ — A1

$\rho = \dfrac{m}{\frac{1}{2}bh} = \dfrac{2m}{bh}$

$\delta m = \dfrac{bx}{h} \cdot \delta x \cdot \dfrac{2m}{bh} = \dfrac{2mx\,\delta x}{h^2}$ — M1

$\delta I_L = \dfrac{1}{3}\delta m\left(\dfrac{l}{2}\right)^2 + \delta m\, x^2$ — M1 M1 A1

$= \dfrac{1}{12}\left(l^2 + 12x^2\right)\delta m$

$= \dfrac{1}{12}\left(\dfrac{4x^2}{3} + 12x^2\right)\dfrac{2mx\,\delta x}{h^2}$

$= \dfrac{80}{36}\dfrac{m}{h^2}x^3\,\delta x$

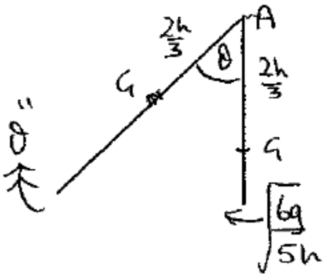
$= \dfrac{20}{9}\dfrac{m}{h^2}x^3\,\delta x$ — A1

$I_L = \dfrac{20}{9}\dfrac{m}{h^2}\displaystyle\int_0^h x^3\,\delta x$ — M1

$= \dfrac{20}{9}\dfrac{m}{h^2} \cdot \dfrac{h^4}{4}$

$= \dfrac{5mh^2}{9}$ * — A1 (9)

(b)



Energy: $\dfrac{1}{2}\cdot\dfrac{5mh^2}{9}\dot{\theta}^2 = mg\dfrac{2h}{3}(1-\cos\theta)$ — M1 A1 A1

$\Rightarrow \dfrac{1}{2}\cdot\dfrac{5mh^2}{9}\cdot\dfrac{6g}{5h} = mg\dfrac{2h}{3}(1-\cos\theta)$ — M1

$\cos\theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{3} \quad (60^\circ)$ — A1 (5)

(c) $M(A)$: $\quad -mg\cdot\dfrac{2h}{3}\sin 60 = \dfrac{5mh^2\ddot{\theta}}{9}$ (max L^r acc^n when at rest) — M1 A1√ on θ

$\left|\ddot{\theta}\right|_{max} = \dfrac{3\sqrt{3}g}{5h}$ — A1 (3)

(17)